Due to ASHRAE's late receipt of this paper, its contents (which includes text, figures, and equations) were not edited according to ASHRAE Special Publications standards.

Estimating Three-Dimensional Below-Grade Heat Losses from Houses Using Two-Dimensional Calculations

Ian Beausoleil-Morrison[1] Gintas Mitalas[2] Harry Chin[1]

ABSTRACT

A technique called the corner correction method *has been developed to estimate three-dimensional below-grade heat losses using two-dimensional calculations. The corner correction method was developed as part of a new approach and software tool, called* BASECALC, *for modelling basement and slab-on-grade heat losses.*

A large number of three-dimensional and two-dimensional numerical calculations were performed for various thermal and geometric characteristics (insulation placement, insulation thermal resistance, soil conductivity, depth, water-table depth, and width). These calculations resulted in corner factors, which can be used to estimate three-dimensional heat losses using two-dimensional calculations.

A series of correlations were developed to express these corner factors as a function of the foundation's and site's thermal and geometric characteristics (insulation placement, insulation thermal resistance, soil conductivity, depth, water-table depth, and width).

The correlations' RMS error for the 1512 corner factors was 1.2%. The greatest deviation between the correlations and a single corner factor was 6.7%. These correlations have been incorporated into BASECALC *to apply the corner correction method.*

INTRODUCTION

Basements and slabs-on-grade account for 10% to 40% of the energy used to heat Canadian houses. Consequently, an accurate and easy-to-use method to model basement and slab-on-grade heat losses is required for determining appropriate insulation strategies and for establishing building- and energy-code requirements.

Starting from the Mitalas method (1982, 1987), a new numerical approach to model basement and slab-on-grade heat losses has been developed. As well, new, easy-to-use software has been created to apply this new approach. The new approach and the software are called BASECALC.

The below-grade portion of the heat transfer from basements and slabs-on-grade is three-dimensional and transient in nature; it would be logical to use a transient three-dimensional method in BASECALC. However, personal computers are not yet fast and powerful enough for this approach and one of the project criteria is to run BASECALC on personal computers.

Rather than transient three-dimensional calculations, BASECALC performs transient two-dimensional calculations. BASECALC then applies a technique to estimate the three-dimensional heat losses based on these two-dimensional calculations. This technique—*the corner correction method*—is the subject of this paper.

DEVELOPMENT OF CORNER CORRECTION METHOD

The Nature of Basement Heat Losses

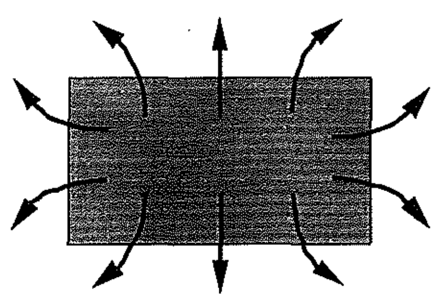

Figure 1 Plan View of Basement with Heat Flow Lines

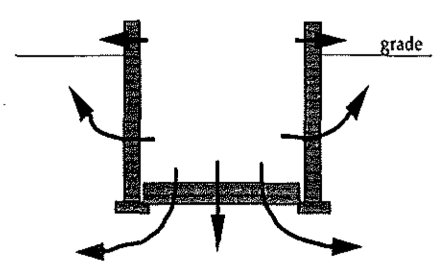

Figure 2 Cross-Section of Basement with Heat Flow Lines

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Figure and Figure illustrate the three-dimensional nature of basement (in this paper, *basement* will be used to mean a basement or slab-on-grade foundation) heat losses. The three-dimensionality is strongest near the corners (see Figure 1) and decreases with increasing distance from the corners. Halfway between two corners, the heat flow becomes two-dimensional (straight arrows in Figure 1).

General Approach

The basement is segregated into notional zones representing these regions of different behaviour (Figure). In the *corner zones,* heat flow is strongly three-dimensional, while in the *central zone,* heat flow is approximated as two-dimensional.

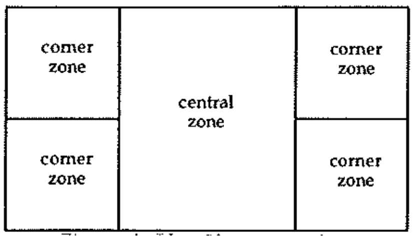


Figure 3 Plan View Showing Heat-Flow Zines

The essence of the corner correction method is to relate the heat loss from the corner zones to the heat loss from the central zone. The *corner factor* is defined as the ratio of the heat loss from the corner zones to the heat loss from the central zone, each normalized its zone's perimeter:

$$F_C = \frac{\left\{\dfrac{q_{\text{corner zones}}}{P_{\text{corner zones}}}\right\}}{\left\{\dfrac{q_{\text{central zone}}}{P_{\text{central zone}}}\right\}}$$

The corner factor is a function of the thermal and geometric characteristics of the basement and site (insulation placement and resistance, soil conductivity, water-table depth, etc.).

A three-dimensional method was used to perform a large number of calculations of corner zones and central zones for various combinations of thermal and geometric conditions. Corner factors were calculated for each case and then correlations developed to relate the corner factor to the thermal and geometric characteristics.

These correlations are the embodiment of the corner correction method, and have been incorporated into BASECALC. BASECALC performs two-dimensional calculations on the central zone; the corner correction method is applied by calculating the corner factors with the correlations; the central-zone results are then adjusted to account for the corner zones using the following equation:

$$q_{\text{total}} = q_{\text{central zone}} + q_{\text{corner zones}}$$

$$= q_{\text{central zone}} \cdot \left\{1 + F_C \cdot \frac{P_{\text{corner zones}}}{P_{\text{central zone}}}\right\}$$

The following sections describe how the corner factors were calculated and how the correlations were developed.

Numerical Modelling

A three-dimensional steady-state program named TRISCO was used to model central zones and corner zones. TRISCO uses an energy-balance technique and a cartesian grid to solve conductive heat transfer problems.

Figure 4 shows how a typical central zone was modelled with TRISCO. Advantage is taken of symmetry by modelling only half the central zone: an adiabat is inserted at the centre of the floor. Another vertical adiabat is placed at a distance from the basement, representing the mid-point to the neighbouring house.

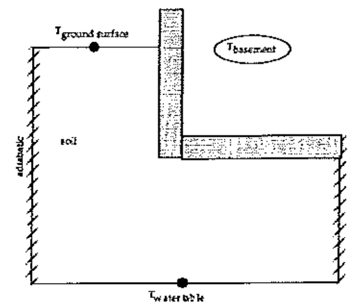


Figure 4 TRISCO Model of Central Zone (Side View)

The ground surface is treated as an isotherm. Another horizontal isotherm is used to represent the water table, the location and temperature of which are assumed to be constant.

Figure 5 and Figure 6 show how a typical corner zone was modelled with TRISCO. In addition to the assumptions made for the central zone, it is also assumed that the interfaces between the central zone and the corner zones are adiabatic.

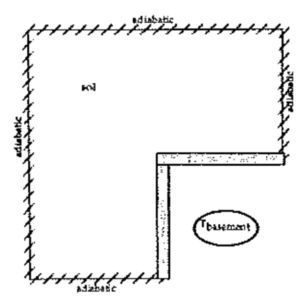


Figure 5 TRISCO Model of Corner Zone (Plan View)

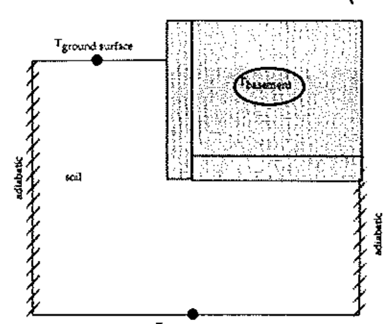


Figure 6 TRISCO Model of Corner Zone (Side View)

Boundary Conditions

The driving forces of the heat transfer are the temperature differences between the basement air and the water table, and between the basement air and the ground surface. In BASECALC, it is assumed that the water-table temperature is constant and that the ground-surface temperature varies sinusoidally over the year. It is also assumed that the water-table temperature is equal to the annually averaged ground-surface temperature.

BASECALC performs separate calculations, using different boundary conditions, to estimate the *average* heat loss and the *variable* heat loss. The *total* below-grade heat loss is the sum of these two components. This is illustrated in Figure .

The average component represents the heat loss from the basement to the water table, plus the annually averaged heat loss from the basement to the ground-surface. BASECALC estimates this component with a steady-state calculation in which the ground-surface temperature and the water-table temperature are set to the same value.

The variable component represents the heat loss from the basement to the ground surface, minus the annually averaged heat loss from the basement to the ground surface. BASECALC estimates this component with a transient calculation in which the ground-surface temperature varies sinusoidally and the water-table temperature is set to the basement-air temperature.

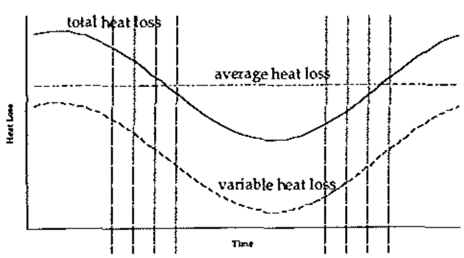


Figure 7 BASECALC's Heat-Loss Components

In the corner correction method, separate corner factors were calculated for the average and variable components of the heat loss. The boundary conditions in the TRISCO runs were set to mimic the boundary conditions used in BASECALC.

Because TRISCO cannot perform transient calculations, the sinusoidal ground-surface temperature could not be prescribed for the variable component. Instead, a constant ground-surface temperature boundary condition was used to model the driving force. This does not perfectly replicate BASECALC's boundary conditions; however, as the corner factor takes the ratio of the corner-zone and central-zone heat losses, any errors will tend to be cancelled.

Variables

Initial runs indicated that the corner factor was most sensitive to the insulation placement, insulation resistance, basement depth, basement width, soil conductivity, and water-table depth. These variables are illustrated in Figure 8.

Figure 8, IP represents the insulation placement (eg. none, interior full-height); RSI represents the thermal resistance of the insulation; D represents the depth (top of floor slab to grade level); W represents the width (exterior of structural wall to exterior of structural wall); V represents the water-table depth (water table to grade level); and k_{soil} is the thermal conductivity of the soil.

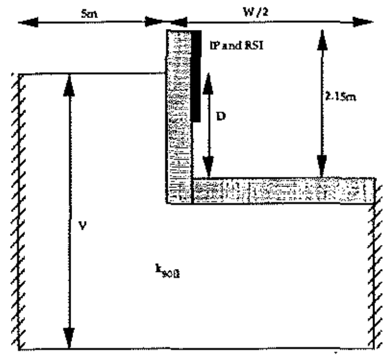


Figure 8: *BASECALC's Heat-Loss Components*

Based on the sensitivity analysis, 756 combinations of these six variables were selected. Central zones and corner zones were modelled with TRISCO for each of these 756 combinations and for each of the two boundary conditions. These 3024 runs required about 60 days of run time on a 486/66Mz computer.

RESULTS

The 3024 TRISCO runs produced 1512 corner factors (one corner factor for each pair of central-zone and corner-zone runs). The distribution of the corner factors are illustrated in Figure 9.

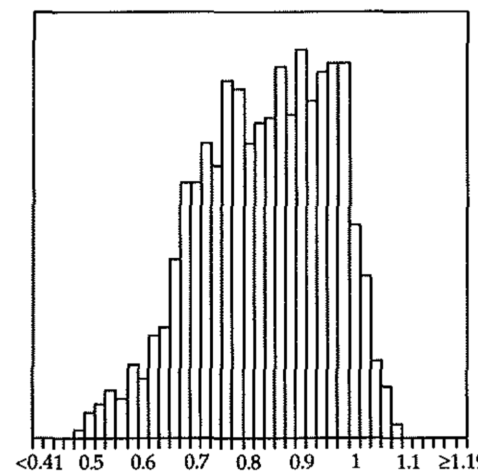


Figure 9 Histogram of 1512 Corner Factors

To simplify application in BASECALC, correlations were sought to relate the corner factors to the thermal and geometric variables. Because they are non-quantifiable, no attempt to correlate the data to the insulation placement (eg. interior full-height) or boundary conditions (ie. average or variable) was made.

It was found that for each combination of insulation placement and boundary conditions, an accurate correlation could be developed between the corner factor and the basement depth, basement width, water-table depth, insulation resistance, and soil conductivity. The same form was used for each of the 16 correlation equations (two boundary conditions/eight insulation placements):

$$
\begin{aligned}
F_C = {} & a + b(RSI) + c(k_{soil}) + d\left(\tfrac{W}{2}\right) + e(D) + f(V) + g(RSI^2) + h(k_{soil} \cdot RSI) \\
& + i\left(\tfrac{W \cdot RSI}{2}\right) + j\left(\tfrac{W \cdot k_{soil}}{2}\right) + k\left(\tfrac{W^2}{4}\right) + l(D \cdot RSI) + m(D \cdot k_{soil}) + n\left(\tfrac{D \cdot W}{2}\right) \\
& + o(D^2) + p(V \cdot RSI) + q(V \cdot k_{soil}) + r\left(\tfrac{V \cdot W}{2}\right) + s(V \cdot D)
\end{aligned}
$$

Where a, b, c ...... s are the correlation coefficients determined by fitting the data.

The coefficients for each of the 16 correlation equations are given in Table 1 and Table 2.

Table 1 and Table 2 also give the root-mean-square (RMS) error and the greatest deviation between the correlation and a data point (the value determined from the TRISCO runs) for each of the 16 correlation equations. The RMS error is less than 2% for 15 of the 16 correlations. The greatest deviation is less than 5% for 13 of the 16 correlations.

The accuracy of the correlations is further illustrated in Figure , which compares the correlations to the data for each of the 1512 corner factors. If the correlations were perfect, all points would lie on the solid line. The RMS error for the full set of 1512 corner factors is 1.2% and the greatest deviation for a single point is 6.7%.

Table 1: Correlation Coefficients

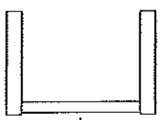
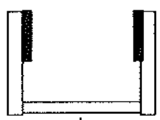
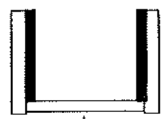
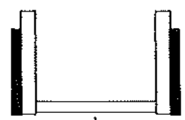
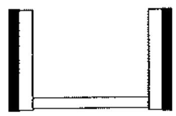
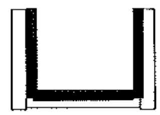
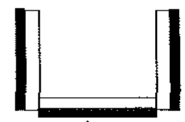
	no insulation		interior insulation from top of wall to 0.6m below-grade		interior full-height insulation		exterior insulation from grade to bottom of wall	
								
	average BC	variable BC	average BC	variable BC	average BC	variable BC	average BC	variable BC
a	0.784	0.756	0.892	0.856	0.717	0.663	0.794	0.753
b	0	0	-0.00590	0.00561	-0.0220	-0.0112	-0.00590	-0.00394
c	-0.0627	-0.0413	-0.0754	-0.0471	-0.0656	-0.0467	-0.0630	-0.0387
d	0.0159	0.0679	-0.0270	0.0393	0.0347	0.104	0.0180	0.0720
e	0.184	0.174	0.0620	0.0733	0.115	0.156	0.162	0.159
f	-0.00280	-0.00143	-0.00249	-0.00260	-0.00243	-0.00100	-0.00371	-0.00103
g	0	0	0.000398	-0.000492	0.00239	0.00123	0.000818	0.000511
h	0	0	-0.000624	-0.00143	0.000171	-0.00139	-0.000322	-7.61E-5
i	0	0	-0.000892	-0.000630	0.000786	0.00145	0.000390	0.000591
j	-0.000588	0.00370	0.00308	0.00552	0.00538	0.00814	-0.000890	0.00414
k	-0.00323	-0.00508	0.00186	-0.00130	-0.00690	-0.0103	-0.00413	-0.00627
l	0	0	0.00237	0.00224	-0.00685	-0.00276	-0.00348	-0.00226
m	0.0123	0.0178	0.0125	0.0151	-0.00593	0.00180	0.00266	0.0142
n	-0.0188	-0.0195	-0.0208	-0.0199	-0.0182	-0.0156	-0.0157	-0.0148
o	-0.0448	-0.0399	0	0	-0.0181	-0.0235	-0.0424	-0.0394
p	0	0	0.000230	-2.58E-5	0.000216	-0.000132	4.03E-5	-7.22E-5
q	0.00116	-0.000694	0.000637	-0.000985	7.41E-5	-0.00130	0.00137	-0.00103
r	0.00186	-0.00235	0.00189	-0.000349	0.00219	-0.000170	0.00206	-0.000156
s	0.00302	0.00259	0.00490	0.00408	0.00495	0.00211	0.00346	0.00205
RMS error	0.9%	0.9%	0.7%	0.5%	1.2%	0.8%	1.0%	0.9%
greatest deviation	2.2%	2.0%	1.5%	1.0%	3.0%	2.3%	2.3%	2.2%
range limits	$0 \le RSI \le 5$ (m^2K/W) $0.85 \le k_{soil} \le 1.9$ (W/mK) $3 \le W \le 10$ (m) $0 \le D \le 2$ (m) $5 \le V \le 15$ (m)							

Table 2: Correlation Coefficients

	exterior full-height insulation		interior full-height insulation and insulation fully covering top of slab		interior full-height insulation and insulation fully covering bottom of slab		exterior full-height insulation and insulation fully covering bottom of slab	
								
	average BC	variable BC	average BC	variable BC	average BC	variable BC	average BC	variable BC
a	0.868	0.860	0.596	0.552	0.674	0.650	0.772	0.777
b	-0.0154	-0.00118	-0.0473	-0.0407	-0.00747	-0.00627	-0.0386	-0.0288
c	-0.0676	-0.0571	-0.0311	-0.0182	-0.0419	-0.0274	-0.0434	-0.0373
d	-0.0347	0.0326	0.0326	0.104	0.0239	0.0885	-0.0346	0.0339
e	0.108	0.168	0.213	0.245	0.137	0.164	0.215	0.260
f	-3.39E-5	-0.000177	0.00203	0.00286	0.00223	0.000988	0.00394	0.00155
g	0.00178	0.000413	0.00500	0.00423	0.00280	0.00209	0.00410	0.00310
h	0.000869	-0.00121	0.00590	0.00407	0.00606	0.00377	0.00543	0.00339
i	0.000392	0.000757	-0.00119	-0.000454	-0.00144	-0.00137	-0.00118	-0.000622
j	0.00834	0.0101	-0.00302	-0.00147	-0.00152	0.000637	1.73E-5	0.00133
k	0.000513	-0.00276	-0.00349	-0.00609	-0.00386	-0.00627	0.00350	0.000809
l	-0.00852	-0.00341	-0.000878	0.00187	-0.00621	-0.00474	0.000134	0.00326
m	-0.00794	0.000214	-0.00436	0.00199	0.00184	0.00404	-0.000678	0.00539
n	-0.0190	-0.0180	-0.0160	-0.0146	-0.0113	-0.00998	-0.0198	-0.0196
o	-0.0133	-0.0218	-0.0379	-0.0465	-0.0250	-0.0307	-0.0387	-0.0503
p	0.000248	-0.000151	-0.000824	-0.00103	-0.000759	-0.000431	-0.000972	-0.00123
q	-0.000442	-0.00123	-0.00140	-0.00224	-0.00147	-0.00210	-0.00200	-0.00247
r	0.00194	-5.57E-5	0.00155	-0.00109	0.00142	-0.000834	0.00152	-0.000763
s	0.00554	0.00222	0.00221	0.000750	0.00275	0.00182	0.00270	0.00121
RMS error	1.3%	2.1%	1.3%	1.5%	1.1%	1.0%	1.4%	1.4%
greatest deviation	3.6%	5.1%	4.2%	6.7%	3.9%	4.5%	3.7%	5.5%
range limits	$0 \le RSI \le 5$ (m^2K/W) $0.85 \le k_{soil} \le 1.9$ (W/mK) $3 \le W \le 10$ (m) $0 \le D \le 2$ (m) $5 \le V \le 15$ (m)							

CONCLUSIONS

A technique called the *corner correction method* has been developed to estimate three-dimensional below-grade heat losses using two-dimensional calculations.

A large number of three-dimensional and two-dimensional numerical calculations were performed for various thermal and geometric characteristics (insulation placement, insulation thermal resistance, soil conductivity, depth, water-table depth, and width). These calculations resulted in corner factors, which can be used to estimate three-dimensional heat losses using two-dimensional calculations.

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